Combinations of Multisets :→

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Let S be a multiset then an r-combination of S is an unordered selection of r objects of S. Thus an r-combination of a multi set S is itself a multi set, a sub multiple of S.

e.g $S = \{2a, 1b, 3c\}$ the 3-combinations of S are

$\{2a, 1b\}$, $\{2a, 1c\}$, $\{1a, 1b, 1c\}$, $\{1a, 2c\}$, $\{1b, 2c\}$, $\{3c\}$

Result :→

① Let S be a multi set with k distinct objects each with infinite repetitions. The number of r-combinations of S is equal to $C(r+k-1, r) = C(r+k-1, k-1)$.

② No. of r-combinations of S = No. of solutions of $x_1 + x_2 + \dots + x_k = r$; $x_i \geq 0$

③ The no. of solutions of $x_1 + x_2 + \dots + x_k = m$; $x_i \geq 1$ is $C(m-1, k-1)$.

④ The no of integral solutions of

$x_1 + x_2 + \dots + x_n = r$ with $x_1 \geq r_1, x_2 \geq r_2, \dots, x_n \geq r_n$

is $C(n-1+r-r_1-r_2-r_3-\dots-r_n, n-1)$.

Q:→ Find the no. of integral solution of the eqn

$$x_1 + x_2 + x_3 + x_4 = 20$$

with $x_1 \geq 3$, $x_2 \geq 1$, $x_3 \geq 0$, $x_4 \geq 5$.

Sol:→ $C(4-1+20-3-1-0-5, 4-1) = C(14, 3)$

OR

Given eqn $x_1 + x_2 + x_3 + x_4 = 20$ —— ①

with $x_1 \geq 3$, $x_2 \geq 1$, $x_3 \geq 0$, $x_4 \geq 5$

Let $y_1 = x_1 - 3$, $y_2 = x_2 - 1$, $y_3 = x_3$, $y_4 = x_4 - 5$
in ①

$$y_1 + 3 + y_2 + 1 + y_3 + y_4 + 5 = 20$$

with $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_4 \geq 0$

i.e. $y_1 + y_2 + y_3 + y_4 = 11$ with $y_i \geq 0$ —— ②

∴ Reqd no. of solution of given eqn ①

= No. of solution of eqn ②

$= C(11+4-1, 11)$

$= C(14, 11)$

$= C(14, 3)$ $\quad (\because C(n, r) = C(n, n-r))$

Q:→ Prove that the number of ways of placing 20 similar books in 5 shelves is $C(24, 4)$.

Sol:→ The reqd. no. of ways is equal to the no. of solutions of the eqn $x_1 + x_2 + x_3 + x_4 + x_5 = 20$; $x_i \geq 0$

$= C(20+5-1, 20)$

$= C(24, 20)$

$= C(24, 4)$

Q:→ Prove that the no. of ways of placing dozen pastaries chosen from 8 different varieties is $C(19, 7)$.

Sol:→ The reqd. no. of ways is equal to the no. of solutions of the eqn $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 12$; $x_i \geq 0$

$= C(12+8-1, 12)$

$= C(19, 12)$

$= C(19, 7)$.

$= C(19,12)$

$= C(19,7).$

$100 < \text{No} < 10,00,000$

$\text{No.} = 101, \ldots, 9,99,999$

Q:→ How many integers between 100 and 10,00,000 have the sum of digits

(i) equal to 5 (ii) less than 5.

Sol:→

Let x_i $(1 \leq i \leq 6)$ be the digit of a number with at most 6 digits

(i) The no. of non negative integral soln of

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 5 \quad ; \quad x_i \geq 0$$

$= C(5+6-1, 5)$

$= C(10,5)$

$= \dfrac{\lfloor 10}{\lfloor 5 \lfloor 5} = \dfrac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$

We want numbers which are greater than 100.

∴ We remove the count of those numbers whose sum of the digits is 5 and less than equal to 100.

(5, 14, 23, 32, 41, 50)

∴ Reqd no. digits btw 100 and 1000000 = 252 − 6 = 246.

(ii) The no. of nonnegative solutions of the eqn

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 5 - k \quad ; 1 \leq k \leq 4$$

with $x_i \geq 0$

is $C(5-k+6-1, 5-k) = C(10-k, 5-k) = C(10-k, 5)$

No. of integers $< 10,00,000 = \sum_{k=1}^{4} C(10-k, 5)$

$= C(9,5) + C(8,5) + C(7,5) + C(6,5)$

$= \frac{\lfloor 9}{\lfloor 5 \lfloor 4} + \frac{\lfloor 8}{\lfloor 5 \lfloor 3} + \frac{\lfloor 7}{\lfloor 5 \lfloor 2} + \frac{\lfloor 6}{\lfloor 5 \lfloor 1}$

$= 209$

We've to remove the integers (positive) which are less than or equal to 100 but sum of digits is less than 5

1, 10, 100
2, 11, 20
3, 12, 21, 30
4, 13, 22, 31, 40

Those are 15 in numbers.

$\therefore$ Reqd no. of integers btw 100 and 10,00,000 which have sum of digits less than 5 $= 209 - 15$
$= 194$

Q:→ Find the numbers of integral solution of

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5$$

with $x_1 \geq -3$, $x_2 \geq 0$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 2$.

Ans $C(19,4)$